Quantum Dynamics of Electron-Nuclei Coupled System in Quantum Dots

Özgür Çakır and Toshihide Takagahara
Department of Electronics and Information Science,
Kyoto Institute of Technology, Matsugasaki, Kyoto 606-8585 JAPAN
and
CREST, Japan Science and Technology Agency, 4-1-8 Honcho, Kawaguchi, Saitama 332-0012, JAPAN
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We have investigated the dynamics of the electron-nuclei coupled system in quantum dots. The bunching of results of the electron spin measurements and the revival in the conditional probabilities are salient features of the nuclear spin memory. The underlying mechanism is the squeezing of the nuclear spin state and the correlations between the successive electron spin measurements. Further we make a proposal for the preparation and detection of superposition states of nuclear spins merely relying on electron spin measurements. For unpolarized, completely random nuclear spin state one can still trace the quantum interference effects. We discuss the realization of these schemes for electron spins on both single and double QDs.

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Electron spins in semiconductor quantum dots(QDs) are considered as one of the most promising candidates of the building blocks for quantum information processing[1, 2] due to their robustness against decoherence effects[3, 4]. In double QD systems, initialization and coherent manipulation of electron spin have been realized, with coherence times extending to 1 μs[5, 6]. Hyperfine(HF) interaction with the host nuclei[7, 8] becomes the main decoherence mechanism, dominating over spin-orbit interactions which act on a timescale of 10s of milliseconds[9, 10] or even longer. Consequently there have been proposals to reduce HF induced decoherence by measuring or polarizing the nuclear spins[11, 12, 13, 14, 15, 16] and to use nuclear spins as a quantum memory[11, 17].

Here we investigate the electron-nuclei spin coupling in QDs, and show that consecutive electron spin measurements following HF interaction are correlated and lead to purification of the nuclear spin system. We predict that the purification of the nuclear spin state would lead to the bunching of results of the electron spin state measurements and also to the reduction in the electron spin decoherence induced by the HF interaction. We will also discuss a strategy for revealing quantum nature of nuclear spins from the correlations of successive electron spin measurements. For the physical realization of the proposals we will in particular discuss a double QD occupied by two electrons, and a single QD occupied by one(two) electron(s).

First of all we consider an electrically gated double QD occupied by two electrons[5, 18]. The excited electronic orbitals of QDs have an energy much greater than the thermal energy and the adiabatic voltage sweeping rates, so that the electrons occupy only the ground state orbitals. Under a high magnetic field, s.t. the electron Zeeman splitting is much greater than the HF fields and the exchange energy, dynamics takes place in the spin singlet ground state $|S\rangle$ and triplet state of zero magnetic quantum number $|T\rangle$. For the singlet state each electron can be found in the different or both in the same QD, whereas for the triplet state electrons can only be found in different QDs. Singlet and triplet states are coupled by the HF fields, and the system is governed by the Hamiltonian,

$$H_e = JS_z + r\delta h_z S_x, \tag{1}$$

where $\mathbf{S}$ is the pseudospin operator with $|T\rangle$ and $|S\rangle$ forming the S_z basis. J is the exchange energy and $\delta h_z = h_{1z} - h_{2z}$, where h_{1z} and h_{2z} are the components of nuclear HF field along the external magnetic field in the first and second dot, respectively. $0 \leq r \leq 1$ is the amplitude of the hyperfine coupling. When both electrons are localized in the same dot, $r \to 0$ and $J \gg \delta h_z$, when they are located in different dots HF coupling is maximized $r \to 1$ and $J \to 0$.

Now we show that by electron spin measurements in a double QD governed by (1), the coherent behavior of nuclear spins can be demonstrated. Electron spins are initialized in the singlet state and the nuclear spin states are initially in a mixture of δh_z eigenstates, $\rho(t=0) = \sum_n p_n \rho_n |S\rangle\langle S|$, where ρ_n is a nuclear state with an eigenvalue of $\delta h_z = h_n$ and satisfies $Tr(\rho_n) = 1$. p_n is the probability of the hyperfine field δh_z having the value h_n. In the unbiased regime $r = 1$, the nuclear spins and the electron spins interact for a time span of τ. Then the gate voltage is swept adiabatically, switching off the HF interaction $r \to 0$, in a time scale much shorter than HF interaction time. Next a charge state measurement is performed which detects a singlet or triplet state. Probability to detect the singlet state is $\sum_n p_n|\alpha_n|^2$, and the triplet state is $\sum_n p_n|\beta_n|^2$ where $\alpha_n = \cos\Omega_n\tau/2 + iJ/\Omega_n \sin\Omega_n\tau/2$, $\beta_n = -ih_n/\Omega_n \sin\Omega_n\tau/2$, with $\Omega_n = \sqrt{J^2 + h_n^2}$. Subsequently one can again initialize the system in the singlet state of electron spins, and turn on the hyperfine interac-

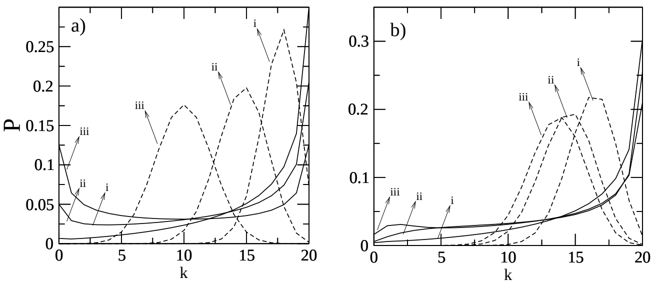

FIG. 1: Probability distribution at $N = 20$ measurements for $k = 0, 1, \ldots, 20$ times singlet detections, for coherent regime(solid lines) and incoherent regime(dashed lines). Two cases of the exchange energy are considered a) $J = 0$ b) $J/\sigma = 0.5$ for HF interaction times $\sigma\tau =$ i)0.5, ii)1.5, iii)∞.

tion for a time span of τ, and perform a second measurement. In general over N measurements, the nuclear state conditioned on $k(\leq N)$ times singlet and $N-k$ times triplet detection is $\sigma_{N,k} = \binom{N}{k}\sum_n p_n|\alpha_n|^{2k}|\beta_n|^{2(N-k)}\rho_n$, the trace of which yields the probability of k times singlet outcomes,

$$P_{N,k} = Tr\sigma_{N,k} = \tbinom{N}{k}\langle|\alpha|^{2k}|\beta|^{2(N-k)}\rangle, \qquad (2)$$

where $\langle\ldots\rangle$ is the ensemble averaging over the hyperfine field h_n[8]. Hereafter, this case will be referred to as the *coherent regime*. One can easily contrast this result with that for the *incoherent* regime in which nuclear spins lose their coherence in between the successive spin measurements and relax to the equilibrium distribution, given by

$$P'_{N,k} = \tbinom{N}{k}\langle|\alpha|^2\rangle^k\langle|\beta|^2\rangle^{(N-k)}. \qquad (3)$$

If the nuclear spins are coherent over the span of the experiment, then successive electron spin measurements are biased to all singlet(triplet) outcomes. In particular, when the initial nuclear spins are unpolarized and randomly oriented, the distribution of hyperfine field is characterized by a Gaussian distribution with variance σ^2, $p[h] = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{h^2}{2\sigma^2}}$ and the summation is converted to an integration, $\sum_n p_n \ldots \rightarrow \int \mathrm{d}h\, p[h]$. As the simplest case, let us check the results of two measurements, each following a HF interaction of duration t. The probability for two singlet detections are given by $P_{2,2} = \langle|\alpha|^4\rangle = \{6+2e^{-2t^2}+8e^{-t^2/2}\}/16$, which is always greater than $P'_{2,2} = \langle|\alpha|^2\rangle^2 = \{4+8e^{-t^2/2}+4e^{-t^2}\}/16$, results given particularly for $J = 0$. As J is increased the probabilities approach each other and for $J \gg \sigma$ they become identical[19].

In Fig. 1, for $N = 20$ measurements, $P_{N,k}$ is shown for HF interaction times $\sigma\tau = 0.5, 1.5, \infty$. For $\tau = 0$, the probability for both (2) and (3) is peaked at $k = 20$. However, immediately after the HF interaction is introduced, the probability distributions show distinct behavior. The measurement results in the incoherent regime approach a Gaussian distribution. In the coherent case the probabilities bunch at $k = 0, 20$ for $J = 0$, and when $J/\sigma = 0.5$ those bunch at $k = 20$ only. As J is increased above some critical value, no bunching takes place at $k = 0$ singlet measurement.

The nuclear spin state conditioned on the previous electron spin measurements is no longer random even if they are initially random. Accordingly, HF induced electron spin decoherence dynamics is also modified. Depending on the results of previous measurement, one may decrease the singlet-triplet mixing. As a particular example, consider the case: Starting from a random spin configuration, N successive electron spin measurements are performed, each following initialization of electron spins in the spin singlet state and a HF interaction of duration $\tau_i (i = 1\ldots N)$ and all outcomes turn out to be singlet. Here the nuclear spin state is given by $\sigma_{N,N}$. Then again HF interaction is switched on for a time t, and the $(N+1)$th measurement is carried out. The conditional probability to detect the singlet state is given by

$$P = \frac{\sum\binom{2}{s_1}\binom{2}{s_2}\ldots\binom{2}{s_{N+1}}e^{-\frac{1}{2}\left[\sum_{i=1}^N(s_i-1)\tilde{\tau}_i+(s_{N+1}-1)\tilde{t}\right]^2}}{4\sum\binom{2}{s_1}\binom{2}{s_2}\ldots\binom{2}{s_N}e^{-\frac{1}{2}\left[\sum_{i=1}^N(s_i-1)\tilde{\tau}_i\right]^2}}, \qquad (4)$$

where the sums run over $s_i = 0, 1, 2$ and $\tilde{\tau}_i = \sigma\tau_i$. For the particular case $\tau_1 = \tau_2 = \ldots = \tau_N = \tau \gg 1/\sigma$, the initial state is revived at $t = n\tau$, $(n = 1, 2, \ldots, N)$ with a decreasing amplitude, $P \simeq 1/2 + \sum_{s=0}^N\binom{2N}{s}e^{\frac{-\sigma^2}{2}(t-(N-s)\tau)^2}/4\binom{2N}{N}$. In Fig. 2 the conditional probabilities(4) are shown for $\sigma\tau = 1.0, 3.0, 6.0$ subject to $N = 0, 1, 2, 5, 10$ times prior singlet measurements in each. Revivals are observable only for $\sigma\tau > 1$, because the modulation period of the nuclear state spectrum characterized by $1/\tau$ should be smaller than the variance σ. The underlying mechanism of revivals is purification of nuclear spins by the electron spin measurements. The purity of a system characterized by the density matrix $\hat{\rho}$ is given by $\mathcal{P} = Tr\hat{\rho}^2$. As an example we are again going to consider the nuclear state prepared by N successive electron spin measurements with singlet outcomes, each following a HF interaction of duration $\tau_1 \ldots \tau_N$. The purity of nuclear spins is given by

$$\mathcal{P} = \frac{1}{\mathcal{D}}\frac{\sum_{s_i=0}^4\binom{4}{s_1}\binom{4}{s_2}\ldots\binom{4}{s_N}e^{-\frac{1}{2}\left[\sum_{i=1}^N(s_i-2)\tilde{\tau}_i\right]^2}}{\left[\sum_{s_i=0}^2\binom{2}{s_1}\binom{2}{s_2}\ldots\binom{2}{s_N}e^{-\frac{1}{2}\left[\sum_{i=1}^N(s_i-1)\tilde{\tau}_i\right]^2}\right]^2}, \qquad (5)$$

where $\mathcal{D}$ is the dimension of the Hilbert space for the nuclear spins. For a fixed ratio of $\tau_1 : \tau_2 : \ldots : \tau_N$, purity (5) is a monotonically increasing function of time. For $\sigma\tau_i \gg 1$, one can attain various asymptotic limits for the purity. For instance, for $N = 2$, there are three asymptotic limits;when a)$\tau_1 = 2\tau_2$ then $\mathcal{P} = 11/4\mathcal{D}$, b)$\tau_1 = \tau_2$ then $\mathcal{P} = 35/18\mathcal{D}$, c)otherwise $\mathcal{P} = 9/4\mathcal{D}$. For $N = 2$ with $\tau_1 = 2\tau_2 = 2\tau \gg 1/\sigma$, the conditional probability

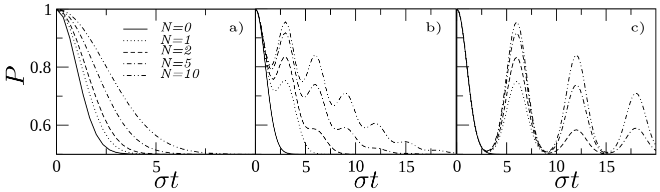

FIG. 2: Conditional probability for singlet state detection as a function of HF interaction time σt, subject to $N = 0, 1, 2, 5, 10$ times prior singlet state measurements and for HF interaction times a)$\sigma\tau = 1.0$, b)$\sigma\tau = 3.0$, c)$\sigma\tau = 6.0$.

(4) is given as, $P \simeq 1/2 + \sum_{n=0}^{3}(4-n)\exp[-(\tilde{t}-n\tilde{\tau})^2/2]/8$ whereas for $\tau_2 = \tau_1 = \tau \gg 1/\sigma$, $P \simeq 1/2 + \{e^{-\frac{(\tilde{t}-2\tilde{\tau})^2}{2}} + 4e^{-\frac{(\tilde{t}-\tilde{\tau})^2}{2}} + 6e^{-\frac{\tilde{t}^2}{2}}\}/12$. It can be seen that as the purity of nuclear spins increases, more revivals are present with an increased amplitude.

So far we have discussed the bunching and revival phenomena only for a double QD system. The same predictions can also be made for a single QD occupied by a single electron[20, 21, 22]. Consider a single QD occupied by a single electron, under an external magnetic field s.t. electron Zeeman energy is much greater than the HF energies. Then the system is described by the Hamiltonian, $H \simeq g_e\mu_B B S_z + h_z S_z$. Here g_e is the electron g-factor, μ_B the Bohr magneton and B is the external field applied in $\hat{z}$ direction. Spin flips are suppressed since $g_e\mu_B B \gg \sqrt{\langle \mathbf{h}^2 \rangle}$. $|\pm\rangle = (|\uparrow\rangle \pm |\downarrow\rangle)/\sqrt{2}$ states are coupled by HF interaction with $|\uparrow(\downarrow)\rangle$ being the eigenstates of S_z. Each time the electron is prepared in $|+\rangle$. Next it is loaded onto the QD, then removed from the QD after some dwelling time τ. Next spin measurement is performed in $|\pm\rangle$ basis. Essentially the same predictions as those for double QD can be made for this system, namely electron spin bunching and revival. We are going to consider electron spin revivals as an example. After N times HF interaction of duration $\tau \gg 1/\sigma$, each followed by $|+\rangle$ measurement, the conditional probability for obtaining $|+\rangle$ in the $(N+1)$th step following a HF interaction of duration t is given as, $P \simeq 1/2 + \sum_{s=0}^{N}\binom{2N}{s}e^{-\sigma^2(t-(N-s)\tau)^2/2}\cos\epsilon[t-(N-s)\tau]/4\binom{2N}{N}$.

The Hamiltonian (1) can also be used to describe a pair of electrons in a single QD[23, 24], and the same predictions as those for a double QD can be made. In the two electron regime, the energy splitting between singlet ground state and triplet excited state can be tuned by application of a magnetic field[24, 25]. Detuning is given by $\Delta E \simeq \omega^2/\Omega$ for $\Omega >> \omega$, Ω being the electron Larmor frequency, and ω is the frequency of the harmonic confinement in the lateral plane. Under a high magnetic field, the triplet state of zero magnetic quantum number is coupled to singlet state via the HF field. This HF field is estimated to be about 0.1 μeV and is comparable to $\Delta E(\simeq 0.2$ meV) for a GaAs QD with 10nm thickness and $\omega = 0.1$ meV under a transverse magnetic field of 20 T. The electrons' spin state can be initialized and measured with high fidelity by a spin selective coupling to leads, relying on spin dependent tunnel rates[24].

In cases so far considered, the quantum nature of nuclear spins is not manifest because the same predictions can be made using semiclassical picture of nuclear spins. In order to detect the quantum behavior of nuclear spins, one has to prepare the nuclear spins in superposition states. For the models under consideration, this can be achieved via switching HF interaction for different components of HF field for successive measurements. This can be realized by changing the direction of the external field. This enables one to observe the interference effects, since different components of the HF field do not commute. As an example we consider the case of a pair of electrons on a single QD with homogeneous HF coupling throughout the dot, i.e. $\mathbf{h} = a\sum \mathbf{I}^{(i)}$ with $a = A/N_n$, N_n being the number of nuclear spins. By applying a magnetic field in $\hat{n}$ direction, s.t., electron Zeeman energy is much greater than the HF fields, an effective HF coupling of the form $V = (\mathbf{h}\cdot\hat{n})(|S\rangle\langle\hat{n};T_0| + \texttt{h.c.})$ can be obtained, with $\hat{n}$ being the quantization axis. The electron is initialized in the singlet state where detection is performed in singlet-triplet basis. We have to perform a series of measurements which involve external field applied in different directions. As a particular example we will consider the conditional evolution of the nuclear system described by $U_c = \mathcal{M}_z U_z(\tau_3)\mathcal{M}_x U_x(\tau_2)\mathcal{M}_z U_z(\tau_1)$, where $U_{\hat{n}}(\tau)$ is the unitary evolution of duration τ following an electron spin initialization in singlet state. $\mathcal{M}_{\hat{n}}$ is the electron spin measurement in $|S\rangle$,$|\hat{n};T_0\rangle$ basis. Initially starting from an ensemble of nuclear spins polarized in $\hat{z}$ direction,

$$\rho(t=0) = \sum_{\lambda,j,m}\frac{P[m]}{F[m]}|\lambda jm\rangle\langle\lambda jm| \qquad (6)$$

where $P[m]$ is the probability of nuclear spins having the polarization $\sum I_z^{(i)} = m$, $F[m]$ is the degeneracy of this subspace. λ enumerates the number of subspaces with the same j. The probability for three singlet detections consecutively along $\hat{z}$,$\hat{x}$ and $\hat{z}$ directions in *quantum mechanical*(QM) picture is given as

$$\sum_{j,k,m,n,n'}\frac{P[m]}{F[m]}\cos^2[am\tau_1/2]\cos[an\tau_2/2]\cos[an'\tau_2/2]$$
$$\times\cos^2[ak\tau_3/2]C^j_{mn}C^j_{mn'}C^j_{k,n}C^j_{k,n'}(F[j]-F[j+1]), \qquad (7)$$

where $C^j_{m'm} = \langle jm'|\exp[-iJ_y\pi/2|jm\rangle$ is the matrix transforming J_z basis to J_x basis in the subspace specified by the magnitude j of the angular momentum operator $\sum \mathbf{I}^{(i)}$ with multiplicity $F[j]-F[j+1]$.

Equation (7) can be contrasted with the *semiclassical*(SC) description of nuclei for which the interference

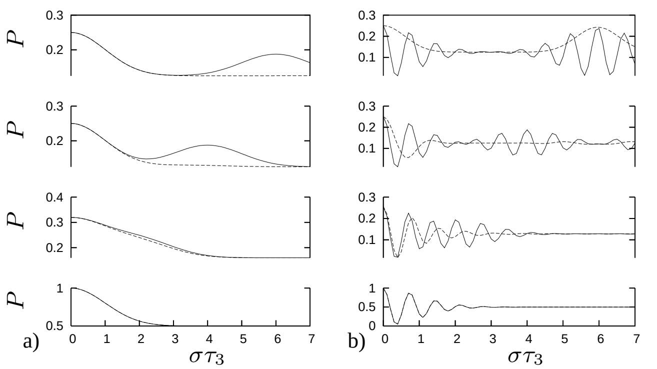

FIG. 3: Probability of three successive singlet detections along $\hat{z}$,$\hat{x}$,$\hat{z}$ directions respectively, for $N_n = 40$ nuclear spins. Initially nuclear spins are polarized along $\hat{z}$ direction with polarizations a) $p = 0$ and b)$p = 0.8$. Solid(dashed) curves correspond to semiclassical(quantum mechanical) picture, and from bottom to top HF interaction times $\sigma\tau_1 = \sigma\tau_2 = 0, 2, 4, 6$ in each graph.

terms are missing, the result of which is

$$\sum_{m,n} P[m]\frac{F[n]}{D}\cos^2[am\tau_1/2]\cos^2[an\tau_2/2]\cos^2[am\tau_3/2],(8)$$

where $I_z^{(i)}$ is a classical Ising spin taking on values $\{-1/2, 1/2\}$ and $D = 2^N$ for a spin 1/2 system. In the semiclassical case the distribution of nuclear HF field along $\hat{x}$ and $\hat{y}$, is random, whereas it is polarized in $\hat{z}$ direction with distribution $P[m]$ as in the QM case.

In Fig. 3, the scheme is exemplified for spin-1/2 nuclei with the number of nuclei $N_n = 40$[26]. Three successive singlet detection probabilities are depicted as a function of τ_3 when the field is along $\hat{z}, \hat{x}, \hat{z}$ respectively. In the SC picture (8), $\hat{x}, \hat{z}$ measurements are independent. The first and the third measurement results, as a function of τ_1,τ_3 respectively, are maximally correlated and this gives rise to revivals as in (4). Whereas in the QM picture (7), these correlations are suppressed due to $\hat{x}$ measurement. Also in Fig. 3b), it is seen that the Overhauser field acting on the electron spin gives rise to Rabi oscillations in the probabilities, which are greatly suppressed in the QM picture. Even in case of unpolarized nuclear spins, SC and QM pictures exhibit distinct behavior. This scheme can also be extended to a a QD with single or a double QD with two electron spins.

Finally, we discuss in brief the feasibility to observe the predicted phenomena. The duration of the cycle involving electron spin initialization and measurement is about 10 μs[5]. Since the nuclear spin coherence time determined mostly by the nuclear spin diffusion is longer than about several tens of ms[27], the bunching for N successive measurements up to $N > 1000$ can be observed. The same holds for the number of revivals that can be observed. For the demonstration of quantum interference of nuclear spins changing the direction of magnetic field before the nuclear spins decohere, may pose some technical difficulties, especially for a single QD occupied by two electrons for which a large magnetic field($\sim$ 10 T) is needed.

In summary, we have investigated the dynamics of the electron-nuclei coupled system in QDs and predicted a couple of new phenomena related to the correlations induced by the nuclear spins. The underlying mechanism is the squeezing and increase in the purity of the nuclear spin state through the electron spin measurements. This squeezing is expected to lead to the extension of the electron spin coherence time because the fluctuation of the nuclear magnetic field due to the dipole-dipole interaction would be reduced. Finally we have proposed a scheme for preparing coherent superposition of nuclear spins based on conditional electron spin measurements. The quantum behavior is manifest even in the case when we have no *a priori* knowledge about the initial nuclear spin state. The predicted results are general and can be confirmed for electron spins on single and double QDs.

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